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Recent research has shown that serious misconceptions frequently survive high school and university instruction in mechanics. It is interesting to inquire whether Newton himself encountered conceptual difficulties before he wrote the "Principia." This paper compares Newton's pre-"Principia" beliefs, based upon his writings, with those of contemporary students in the areas of impetus force and centrifugal force. The paper emphasizes the retarding effect on Newton's development of inappropriate but deep-rooted models at a qualitative level, and the extent to which his experience suggests the necessity for students to struggle conceptually in order to construct the models employed by physicists. Lists 9 references. (YP)

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GENIUS IS NOT IMMUNE TO PERSISTENT MISCONCEPTIONS: CONCEPTUAL DIFFICULTIES IMPEDING ISAAC NEWTON AND CONTEMPORARY PHYSICS STUDENTS

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ABSTRACT

Recent research has shown that serious misconceptions frequently survive high school and university instruction in mechanics. It is interesting to inquire whether Newton himself encountered conceptual difficulties before he wrote the Principia: (a) Did he have serious difficulties? (b) If so, were they difficult to overcome? We shall present evidence from Newton's writings of affirmative answers to both questions.

Newton's development of his system of mechanics was hampered by a persistent belief in "the force of a [moving] body" from 1664 to 1685. His belief in centrifugal force was an additional restraining factor that remained intact until Hooke's intervention in 1679 and weakened only gradually over the next two years. Three additional years passed before the resulting successes weakened his commitment to impetus sufficiently to permit conceptualization of mass as an inert surrogate.

This paper will compare Newton's pre-Principia beliefs with those of contemporary students in the areas of impetus force and centrifugal force. We shall emphasize the retarding effect on Newton's development of inappropriate but deep-rooted models at a qualitative level, and the extent to which his experience suggests the necessity for students to struggle conceptually in order to construct the models employed by physicists.

INTRODUCTION

Studies of physics students' reasoning about the relationship of force to motion have demonstrated that they harbor misconceptions which can interfere with learning and which are surprisingly resistant to change. Consider, for example, the following data from high school and university students in typical settings:

(1) Among a high school group asked to compare force magnitudes exerted when a bowling ball strikes a bowling pin, only 5% thought the pin exerts as much force on the ball as conversely. [Brown and Clement (1987)] These and the data below are from questions asked after instruction in mechanics had been completed.

(2) Only 29% of university physics students correctly drew the single, downward force of gravity acting on an ascending coin tossed in the air. In 90% of the incorrect answers the students drew force vectors acting in the direction of motion that are fictitious from the physicist's point of view but are strongly reminiscent of the impetus beliefs of medieval theorists. [Clement (1982)]

(3) Only 7% of high school physics students correctly indicated the single, centripetal force acting on a stone twirled horizontally over one's head in horizontal circular motion (ignoring gravity and air resistance). 71% indicated there would be an outward centrifugal force acting on the stone, 67% that there would be a force in the direction of motion of

the stone, and 48% that both of these forces would be acting on the stone. (New data: sample size = 87)

The beliefs exhibited in these examples are sources of difficulty at the level of basic conceptual understanding of the material. Their existence suggests that the concepts instructors are trying to impart are being misunderstood at a qualitative level, in addition to any difficulties that arise with quantitative formulation. What we wish to discuss in this paper is the extent to which Newton's thinking was hampered by similar inappropriate conceptions, and the extent to which he had to struggle to free himself from them.

The conceptual framework for dynamics which Newton set forth in his Principia is based on the assumption that the effect of a force acting on a material object is to change the speed and/or direction of motion. The immense success of this framework during the last three centuries has made its usefulness to mature professionals very clear. For the uninitiated, however, there is a problem of comprehension: The framework can be made one's own only if the possibility of motion without force is accepted. That state of mind is very difficult for students to achieve, because of a deep-rooted conviction that motion implies a force that causes it. [Clement (1982)]

Newton's writings before the Principia reveal the same problem, and the important question for us is: Why did he take so many years of intermittent and often intensive effort to

overcome it? We have been intrigued by the evidence that Newton's progress was blocked by a web of interconnected misconceptions. Impetus was the key barrier that he had to overcome in order to write the Principia, with several others playing strong supporting roles. We feel that the long term persistence of this major misconception in so gifted a person says something important about the magnitude of the cognitive change that students must negotiate with themselves in order to understand Newtonian mechanics. Bringing the details to light may yield insights that can benefit the design of instruction. Conversely, research on students' cognitive difficulties may help make Newton's developmental path more intelligible.

There are at least four ways in which parallels might be drawn between Newton and present-day students with regard to conceptual difficulties: (a) the specific qualitative misconceptions; (b) whether these significantly impede progress; (3) how they impede progress, if in fact they do; and (4) how these difficulties are overcome. In this paper we focus on the second question and touch on the first and fourth questions, leaving the third and its overlap with the others for future papers. To anticipate our conclusion: We find that Newton's progress was indeed significantly hampered by qualitative conceptual difficulties that strongly parallel those found in students. We also discuss how, with the help of others, he finally broke through to the profound insights embodied in the Principia.

THE PERSISTENCE OF FORCE AS PROPERTY

What Newton called "the force of a body" is the intuition that a moving body is a reservoir of force. Our investigations of students' misconceptions indicate that this idea can have two aspects:

IMPETUS -- the idea that motion which occurs without assistance from an external propelling agent is sustained by an internal motive force

TRANSFER -- the idea that one body may give up some of its impetus force to another during impact

As an example of the transfer misconception in a student before instruction, consider the following reply to a question about a moving ball striking a stationary ball: "The force from the moving ball would be transferred to the stationary ball, so the force would move from the moving ball to the ball that wasn't moving." Another student said: "...the only force that would be transferred would be that which the stationary one could absorb before it moves..."

The persistence of transfer after instruction can be seen in the following data. When given the choices below for describing the impact of a cue stick on a cue ball, 65% of high school physics students selected choice (1):

(1) The cue stick transfers force to the cue ball.

(2) The cue stick exerts a force on the cue ball,
but does not transfer force to the cue ball.
(New data: sample size = 87)

The invention of impetus seems to be a response to the mindset that motion without the intervention of externally applied force requires an explanation. The impetus idea is used in the transfer idea: Collisions can be perceived as providing evidence for transfer, and belief in transfer supports belief in impetus.

The earliest evidence for the transfer-of-impetus conception in Newton appears in a notebook dated 1664, when he was 21 years old and in his third year at Cambridge University, in a passage titled "On Violent Motion" in which he employs transfer to explain projectile motion: [Herivel (1965) p.123]

> This motion is not continued by a force impresst
> [from the outside] because the force must be
> communicated from the mover into the moved ....

To our knowledge, the transfer aspect has not been explicitly discussed in previous descriptions of Newton's conceptual development.

The above statement is very much in the spirit of Descartes, whom Newton had been reading. We interpret the agreement with Descartes as making intuitive sense to the young Newton, rather than as assent to authority, because the belief expressed here persisted for two decades despite continuing

serious disagreement with Descartes about relativity of motion. "On the issue of [Descarte's] relativism", writes Westfall, "which smacked of atheism in Newton's view, he continued to shout his defiance until his dying day." [Westfall (1980) p.415]

Newton made considerable headway against the impetus idea in 1665-66 by focussing on relationships involved in the impact of two bodies. However, this progress was soon followed by a cripling setback when he reimbraced the impetus concept. In this section we describe these periods of progress and relapse in Newton's thinking.

In a passage in the Waste Book titled "Of Reflections", he recognized that changes of speed and direction are dynamically important, and that these changes are effectuated by forces originating outside the affected body. [Westfall (1980) p.146]

> Soe much force as is required to destroy any quantity of motion in a body soe much is required to generate it; & soe much as is required to generate it soe much is alsoe required to destroy it.
>
> .... equall forces shall effect an equall change in equall bodies.... For in loosing or getting ye same quantity of motion a body suffers ye same quantity of mutation in its state, and in ye same body equall forces will effect a equall change.

As part of this promising elaboration of the force concept, Newton also explored the two-body interaction forces required to maintain the whole-system center of mass in motion at constant velocity. "Examining what he called 'ye mutual force in reflected bodies', he concluded that each one acts equally on the other and produces an equal change of motion in it. To those who had tried to understand impact through the force of a body's motion, this conclusion had appeared patently false.... Newton appears to have reached it by realizing that every impact can be viewed from the special frame of reference of the common center of gravity of the two bodies.... Newton never forgot this conclusion. It contained the first adumbration of his third law of motion..." [Westfall (1980) p.147]

With good beginnings toward the second and third laws at age 22, what about the first law? In a cursory reading it is easy to imagine that Newton had also arrived at the principle of inertia: [Westfall (1980) p.145]]

> Every thing doth naturally persevere in ye state in which it is unlesse it bee interrupted by some externall cause.
>
> [A] body once moved will always keepe ye same celerity, quantity and determination of its motion.

Statements like these plainly deemphasize internal force compared to external, but they do not rule out the continued presence of a latent impetus concept.

Westfall points out the developmental importance of Newton's early realization that force must be associated with changes of velocity: "Here was a new definition of force in which a body was treated as the passive subject of external forces impressed upon it instead of the active vehicle of force impinging on others." However: "All of the possibilities inherent in the insight did not appear immediately to the young man who had earlier accepted the idea of a force internal to bodies which keeps them in motion. It is clear to us that the earlier notion was incompatible with the principle of inertia and his new conception of force. It was not at first equally clear to Newton. Instead of rejecting the idea of internal force outright, he attempted to reconcile it with the new concept he was developing." [Westfall (1980) p.146]

What happened, in our view, is that Newton began to interpret force exerted from the outside as force transfer: [Westfall (1980) p.146]

> The force wch ye body (a) hath to preserve it selfe in its state shall bee equall to the force wch [pu]t it into yt state; not greater for there can be nothing in ye effect wch was not in ye cause nor lesse for since ye cause only looseth its force onely by communicating it to its effect there is no reason why it should not be in ye effect wn tis lost in ye cause.

We shall return later to the question of why Newton retreated from externally impressed force to force transfer. For the present, we wish only to point out that impetus did indeed remain a viable concept in Newton's mind for another two decades. This is clearly articulated in the manuscript De Motu which he sent to Edmond Halley in the fall of 1684, much of which later found its way into the Principia. The translation from the Latin reads: [Westfall (1980) p.411]

> I call that by which a body....endeavors to persevere in its motion in a right line the force of a body or the force inherent in a body.

> By its inherent force alone, every body procedes uniformly in a right line to infinity unless something extrinsic hinders it.

Note that Newton wrote these lines after his pioneering work of 1680 in which (a) at Hooke's suggestion he began to think of the Sun as exerting an attractive force on a planet, conceived as an action that continually deflects the latter's motion from the straight line it would otherwise travel, and (b) returned to his idea of 1665 that force is associated with changes of speed and direction. In this work he was able to identify major features of the force responsible for planetary motion: (1) Kepler's equal areas pattern of orbital motion was found to be a consequence of the force being directed always toward the Sun. (2) An inverse square variation of force magnitude with distance from the Sun was inferred from the elliptical form of the orbits. In spite of this tremendous progress, he was not ready to give up impetus for another five years.

Of course, a person with Newton's powerful drive for conceptual coherence could not continue indefinitely to ride two horses in different directions. His meeting with Halley in the spring of 1684, at which he made known his deduction of the inverse square law of attraction of the planets by the Sun, ushered in a period of intense cognitive change. De Motu, which was sent to Halley after that meeting, was itself revised and then followed by two shorter papers of revisions. [Herivel,

documents X(a) and X(b)] These papers reveal that Newton was thinking quite differently by the spring of 1685.

The first short paper reveals a newly acquired ability to think of rest and motion as equivalent: [Herivel, p.311]

> The inherent and innate force of a body is the power by which it preserves in its state of rest or of moving uniformly in a straight line.
>
> Force impressed on a body is that by which a body is urged to change its state of moving or resting...

The second short paper speaks with greater precision: (a) "The inherent and innate force of a body" becomes "The internal force of matter". This "internal force" is not impetus, however. Newton explains that it is [Herivel, p.318]

> the power of resistance by means of which any one body continues so far as it can in its state of rest or moving uniformly in a straight line...nor differs at all from the inertia of matter except in our mode of conceiving it.

He goes on to differentiate the new conception of "internal force" sharply from impetus:

> In fact a body only invokes this force in changes of state produced in it by another force impressed on it...

This novel idea of a force that acts only during a change of motional state appears to be a rudimentary conception of mass as an inert surrogate for impetus. Westfall describes it as "a paradoxical conception of a matter both inert and active". The vestigial use of impetus-like terminology for inertial mass persisted into the Principia as vis inertiae. [Footnote 3]

With impetus in the process of being transformed into inertial mass, the second short paper goes on to explain what is meant by "impressed force". It is

> an action excercised on a body to change its state of rest or motion. This force consists truly in the action only, nor does it remain in the body after the action.

The conception of impressed force as "action only" appears to signal Newton's victory over the intuition of transfer, and that of force as something which does not "remain in" a body seems to represent his final triumph over impetus. Force has now become something that influences motion only from the outside. A complete separation of force and motion has finally been achieved which, together with a budding conception of inertial mass, makes it possible to contemplate motion without force.

Newton set out in 1665 to modify his early conception of force as a property associated with motion into a form suitable for expressing action by one body on another. Why did he cling to the impetus force idea for so many years thereafter? Why

were the principle of inertia and the concept of inertial mass the last of his successes? We feel the answer lies in a deep-rooted conviction, derived from a lifetime of kinesthetic experience, that motion requires a causal explanation and that "the force of a body" is a reasonable description of the cause. Barring the way to a conception of force as an action from without was Newton's commitment to impetus and to centripetal force. The latter was linked to his belief in impetus, and appears to have played a strong reinforcing role.

IN THE GRIP OF CENTRIFUGAL FORCE

In his article on Newton's discovery of gravity in 1685, Cohen writes that Newton "had more or less accepted the inertia principle some 20 years earlier." But his progress in the direction of constructing a more adequate conception of force was halted because "Newton, like Descartes and Huygens, was so mired in the concept of centrifugal endeavor that the full implications of inertial physics were far from obvious to him." [Cohen (1981)]

Practically everyone senses, on the basis of personal kinesthetic memory, that a body revolving in a circle strives to push itself away from the center of its circular path -- and to do so more strongly when it is moving at greater speed. This palpability of centrifugal force could have reinforced Newton's

belief in impetus by providing a second class of situations in which it seems reasonable to think of force as a property of a moving body. [Footnote 2] Westfall writes: "Following both Descartes and common experience, Newton agreed that a body in circular motion strives constantly to recede from the center, like a stone pulling on a string as it is whirled about. The endeavor to recede appeared to be a tendency internal to a moving body, the manifestation in circular motion of the internal force which keeps a body in motion." [Westfall (1980) p.148]

For Newton, there also appear to have been some rational aspects of the problem that were peculiar to himself: (a) During the same period (1665-66) in which he began to think of force as associated with changes of speed and direction caused by impact, he derived the centrifugal force law by considering elastic collisions of a moving body with the sides of a motionless polygonal cavity and letting the number of sides become infinite. (b) Moreover, as Westfall points out, belief in the reality of centrifugal force is less easily shaken by relativistic considerations than is belief in impetus force because: "Unlike the internal force of rectilinear motion, it could not be made to disappear by shifting inertial frames of reference." These conclusions could have reaffirmed Newton's waning belief in impetus force (a) through association with centrifugal force and (b) through confidence in the absoluteness of the latter.

In Westfall's summary judgement: "Every indication from Newton's papers suggests that the problems of circular motion, together with considerations external to mechanics, soon led him to reject the principle of inertia. Twenty years later, the same problems of circular motion viewed from a new perspective would be decisive in his final conversion to inertia."

Newton assumed that the centrifugal force of a planet moving in a circle is balanced by a force of external (and obscure) origin which is directed toward the Sun. Some time in the late 1660's, he substituted Kepler's third law into his formula for centrifugal endeavor and discovered that [Turnbull (1960) vol.I p.297 and vol.III p.46]

> ...for the primary planets, the endeavors of receding from the Sun will be reciprocally proportional to the square of their distances from the Sun.

This spurious quantitative success may have offered Newton temporary conviction that his balanced force model was on the right path. There is evidence as late as 1673 for Newton's hope that his finding would be worthy of professional recognition: After Huygens published the centrifugal force formula earlier in that year, Newton wrote to Henry Oldenburg, for transmission to Huygens, about his prior application of the formula. [Turnbull (1960) vol.I p.290]

The end of Newton's wandering in the desert of centrifugal force came in November 1679 when Robert Hooke, in his capacity as secretary of the Royal Society, wrote to Newton [Turnbull (1960) vol.II, p.297] inviting his opinion on Hooke's idea of

> ...compounding the celestiall motions of the planetts [out] of a direct motion by the tangent & an attractive motion towards the central body.

Newton replied immediately that he did not

> ...so much as heare (yt I remember) of your Hypotheses of compounding ye celestial motions of ye Planets of a direct motion by the tangt to ye curve...

Westfall remarks that "in a letter to Halley of 20 June 1686, he spoke of Hooke's Lectures and Collections (1678) as though he had read them when they appeared." [Westfall (1980) p.383] Thus there is evidence that Newton had actually known of Hooke's proposal for more than a year but had failed to act on it. We attribute this inaction to the continuing strength of his commitment to centrifugal force.

But Hooke's letter did indeed start Newton on the way out of his doldrums. Why was 1679 different from 1678? We suggest that the intervening year had moved Newton to a state of greater receptiveness because his encounters with alchemy had by then aroused in him a previously nonexistent capacity to imagine

distant "attractions" and "repulsions" that exert force without impact. Westfall writes: "Driven, as it seems to me, primarily by the phenomena that he observed in alchemical experimentation, and encouraged by concepts that he encountered in alchemical study, Newton appeared to be poised on the brink of a further break with the mechanical philosophy, which would have major impact on his future career." [Westfall (1980) p.377]

In the course of followup correspondence with Hooke in 1680, Newton made his first progress toward a model of orbital motion employing unbalanced forces. But he did this without actually giving up centrifugal force. He argued that a body moving under the action of a central force will [Turnbull (1960) vol.I p.307]

> ...circulate wth an alternate ascent and descent made by it's VIS CENTRIFUGA & gravity alternately overbalancing one another.

A precipitous discarding of centrifugal force as a viable concept in dynamics was clearly impossible for Newton. As late as April 1681 he was continuing to experiment with its usefulness in analyzing the trajectory of a comet near the Sun. Writing to John Flamsteed, he spoke of [Turnbull (1960) vol.II p.358]

> ...the VIS CENTRIFUGA overpow'ring the attraction & forcing the Comet there notwithstanding the attraction, to begin to recede from ye sun.

By the time he wrote De Motu in 1684, Newton had permanently abandoned centrifugal force in discussions of orbital motion. Even in the Principia, however, he continued to employ it when discussing rotating liquids. [Herivel (1965) p.57]

Cohen calls Hooke's intervention "A major preliminary to universal gravity.... Newton himself said more than once that it was the correspondance with Hooke during the years 1679 and 1680 that really set him thinking fruitfully in a new way about the cause of celestial motions." [Cohen (1982)] Even so, Newton was not able to assimilate Hooke's conception all at once. Rather, his willingness to adopt Hooke's point of view appears to have grown only by degrees over the next few years, as the growth of successes using that point of view made its superiority ever more clear.

Making Hooke's conception his own required two changes in Newton: (1) A conceptual reorientation in which he set aside the centrifugal force idea and abandoned the balanced force model of circular orbits. (2) A procedural reorientation in which he learned to superpose inertial motion with motion due to an attractive force. These advances combined with Newton's earlier association of force with changes of velocity and with a new-found ability -- acquired only during the late 1670s -- to accept the possibility of action at a distance. All this made it possible for him to demonstrate enough important connections between his emerging dynamics and Kepler's laws to make him ready for the final stimulus of Edmond Halley's visit in 1684.

CONCLUDING REMARKS

In this paper we have attempted to document certain of Newton's qualitative conceptual difficulties, and to show that these presented major obstacles in his path to the Principia. But granted that these difficulties successfully impeded the progress of one of history's preeminent minds, one might still be tempted to argue that they should be of little concern in contemporary physics teaching because: (1) Newton lived in an intellectual climate in which impetus beliefs were widely held by his peers. (2) Newton had no teacher to tell him the correct perspective. We shall argue against both of these considerations.

With regard to the first point, there is strong evidence from numerous studies of contemporary students that Newtonian ideas have not filtered down to the general intellectual climate of which students are a part. In their interaction with peers who have not taken physics (and with some who have), students are faced with many of the same conceptions that Newton faced in his peers.

With regard to the second point, although Newton had no teacher in the narrow sense, he did experience strong positive outside influences, particularly from Hooke. Even when Hooke's letter pointed him in the right direction, however, Newton's prior conceptions were powerful enough to prevent his moving rapidly to a more adequate way of analyzing planetary trajectories. He needed more than five years to work through

the full implications of Hooke's dynamical model.

Students appear to be impeded by their preconceptions in a similar way. A wealth of data demonstrates that simply being told about more adequate conceptions often does not have a lasting effect on their reasoning. Students appear to need to confront the conflict between old and new views directly in order to make progress. Newton's experience suggests that struggling with peer confrontation is an important part of this process, and moreover that considerable time will be required for this interaction to work its beneficial effect.

We feel that the mutual support provided by a web of interconnected misconceptions goes a long way toward explaining persistance in Newton and in students. To dismantle a system of misconceptions requires something like a revolution in one's world view, which can only be precipitated by conceptual conflict. In Newton's case, the collusion of centrifugal force and impetus force to form a system with a significant degree of coherence makes it understandable why an upheaval in his world view was necessary in order to produce the Principia. We plan to address coherence in students on another occasion.

We would like to raise one final issue in a tentative manner, as a question to be discussed. Might the memory of a long personal struggle help explain why Newton elected to designate the principle of inertia as a separate fundamental principle in the Principia? Since the first law associates zero force with constant velocity, he had the option of regarding it

as being subsumed by the second law. Why then did he not discard it, as he did a number of other candidates for the status of fundamental principle which he had considered in his earlier writings? It is intriguing to consider the hypothesis of a feeling on Newton's part that to do so would have obscured a conceptual issue which had been developmentally important for him. Perhaps it will be possible for science historians to shed some light on this question.

FOOTNOTES

1. In this paper we use the term "misconception" to designate any conception that is in conflict with classical mechanics as it is understood today.

2. We intend to explore in a future paper how the centrifugal force and impetus force ideas may have interacted and been mutually supportive for Newton.

3. See Science, vol.176, p.157 (1972) for a review of Westfall's Force in Newton's Physics which argues that such terminology indicates Newton retained a true impetus concept through the first two editions of the Principia.

We wish to avoid attaching any negative connotation to the term since such naive conceptions can be complex, creative and adaptive constructions of the mind.

REFERENCES

David E. Brown and John Clement, in Joseph D. Novak ed., Proceedings of the Second International Seminar on Misconceptions in Science and Mathematics, Cornell University (1987)

John Clement, American Journal of Physics, vol.50, p.66 (1982)

I. Bernard Cohen, Scientific American, vol.245, p.166 (1981)

I. Bernard Cohen, in Zev Bechler ed., Contemporary Newtonian Research, D. Reidel Publishing Company (1982)

John Herivel, The Background to Newton's Principia: A Study of Newton's Dynamical Researches in the Years 1664-1684, Oxford University Press (1965)

Herbert W. Turnbull, The Correspondance of Isaac Newton, Cambridge University Press (1960)

Richard S. Westfall, Force in Newton's Physics, American Elsevier Publishing Company (1971)

Richard S. Westfall, The Construction of Modern Science, Cambridge University Press (1977)

Richard S. Westfall, Never at Rest: A Biography of Isaac Newton, Cambridge University Press (1980)